Privacy Enhancing Technologies (GA17)
Modern privacy-friendly computing

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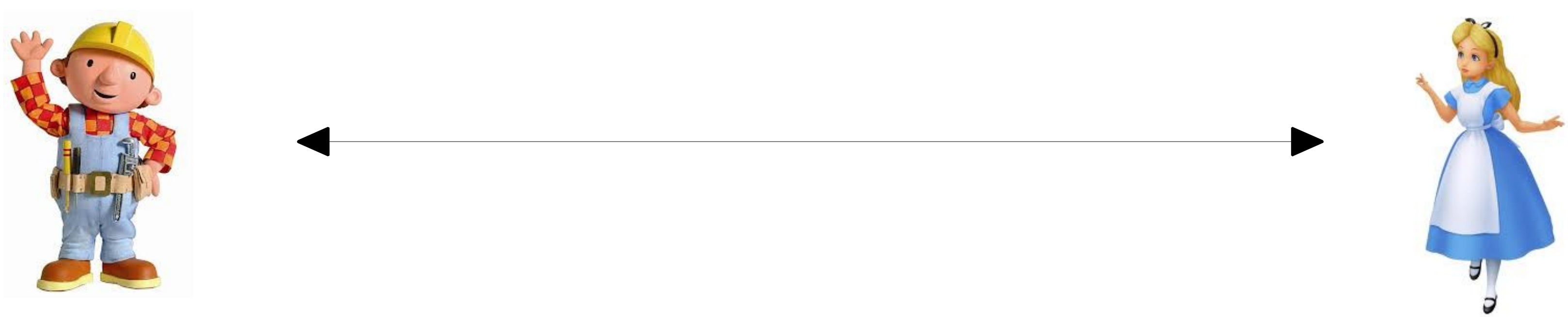
The "easy" privacy problem: Hiding information from third parties



- Alice and Bob trust each other with their "private" information.
- They wish to hide their interactions from third parties:
  - Encryption hides content.
  - Anonymous communications hide meta-data.
- A relatively well-understood problem.
  - Widely deployed (TLS, Tor).

The “hard” privacy problem: Hiding information from your partners

Who is richer?

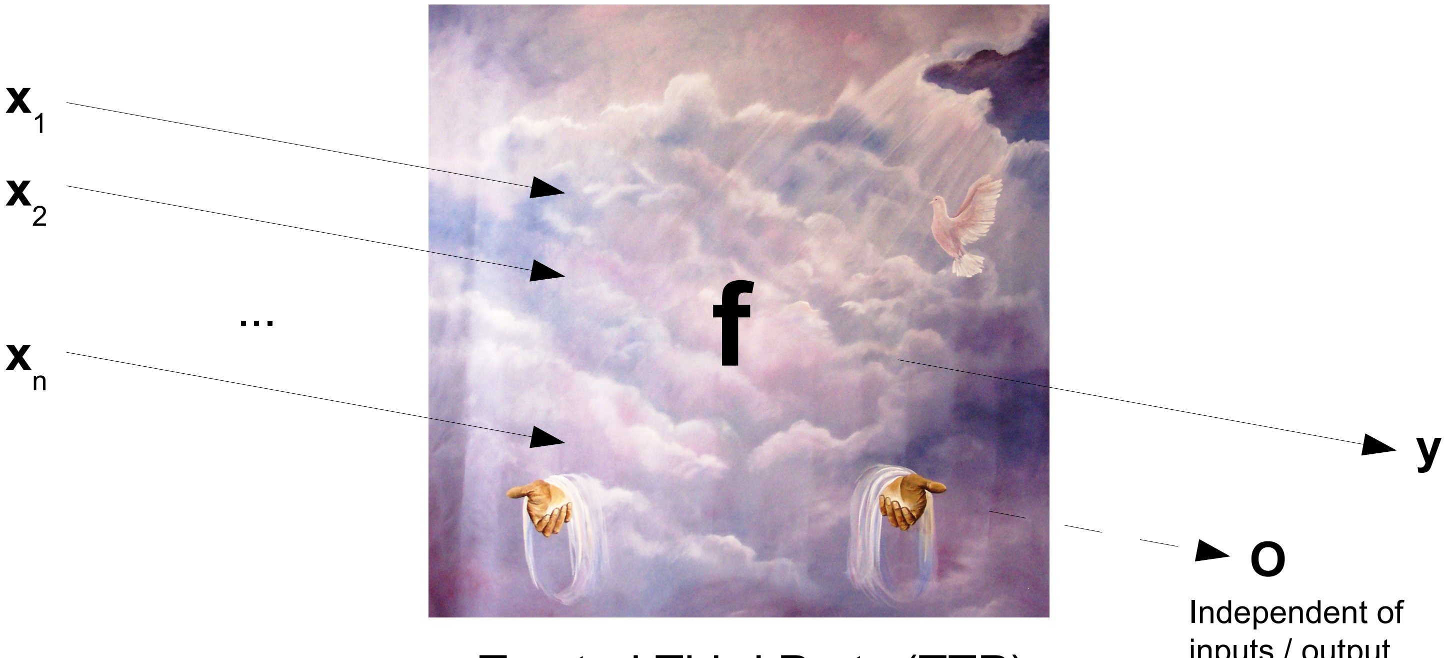
I am also curious but I do not want to tell you how much I earn.

- Example: “The Millionaire's problem” (Yao)
- Alice and Bob do not trust each other with their secrets, but still want to jointly compute on them.
- Associated problem: they may not trust each other to perform any computations correctly.

The formal problem

- Consider a function **f** with **n** inputs $\mathbf{x}_i$ from distinct parties returning a result: $\mathbf{y} = \mathbf{f}(\mathbf{x}_1, \ldots, \mathbf{x}_n)$
  - Correctness: We want to compute **y**
  - Privacy: do not learn anything more about $\mathbf{x}_i$ than what we would learn by learning **y**. Despite the observations **o** from the protocol
- In terms of probability:
  - $\Pr[\mathbf{x}_i \mid \mathbf{o}, \mathbf{y}, \mathbf{x}_j] = \Pr[\mathbf{x}_i \mid \mathbf{y}, \mathbf{x}_j]$

Straw-man Solution: Trusted Third Party



Trusted Third Party (TTP)

TTP: Every participant has to trust TTP for confidentiality, integrity and availability.

What is wrong with Trusted Third Parties

- May not exist!
- Even if it may exist: The 4 Cs
  - **Cost**: what is the business model? How to implement cheaply?
  - **Corruption**: How do you really know that it will not side with the adversary?
  - **Compulsion**: Legal or extra-legal compulsion to reveal secrets.
  - **Compromise**: It may get hacked!
- Conclusion:
  - TTP: not a robust implementation strategy.
  - However: a great **specification strategy** (ideal functionality).

Theory:
"Any function can be computed privately without a TTP"

- Even without a coordinator.
- Participants do not learn other's secrets.
  - Can be made robust to cheating.

- Two adversary models:
  - Honest but curious: adversary executes protocols correctly but tries to learn as much as possible. (½ N + 1 honest)
  - Byzantine: will send, or drop arbitrary messages to learn the secrets. (2/3 N +1 honest)
- Both can be tolerated, but with different efficiency.

How does one prove this generic result?

A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

- Computation theory:
  - NAND is sufficient to represent any boolean circuit.
  - NAND can be expressed using the algebraic expression:

$$\mathrm{NAND}(A,B) = 1 - AB$$

  - If we can express binary digits, compute addition and multiplication privately, we can compute all circuits.

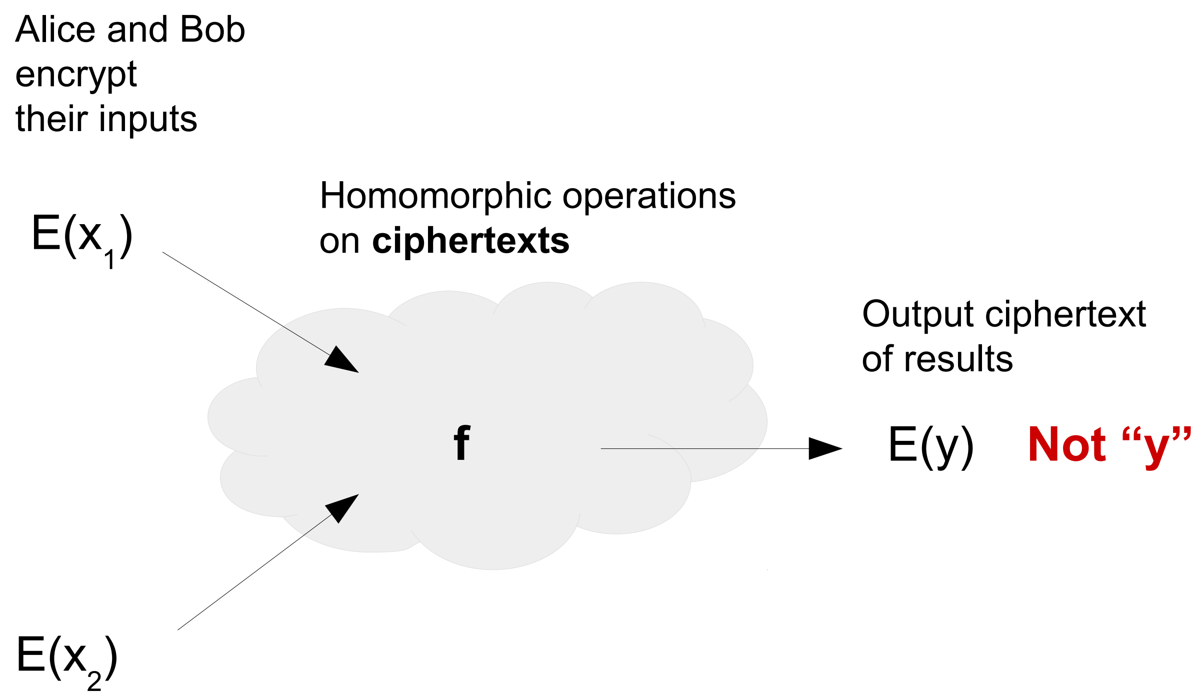
Two approaches

- Homomorphic encryption:
  - Express 0,1 as randomized ciphertexts E(0), E(1).
  - Allow for operations on ciphertexts producing the cipher text of an addition and multiplication.
  - Here in depth: additive homomorphism only.

- Secret sharing:
  - Express 0,1 as “shares” distributed between users.
  - Do addition and multiplication using protocols on shares.
  - Here in depth: SPDZ addition and multiplication.

Homomorphic Encryption

Homomorphic encryption
The Big Picture



Additively homomorphic public-key encryption

- Goal – define functions for:
  - GenKey
  - Encrypt
  - Decrypt
  - Add
  - (no multiply)

- Note:
  - Add n times is *multiplication with a public constant*

Mathematical reminder

- Define two elements g,h that are generators of a cyclic group within which the discrete logarithm problem is believed to be hard.
  - Generators means: g^i may lead to all group elements.
  - Discrete logarithm problem:
    - Given g, $x \rightarrow g^x$ is easy to compute.
    - Given g, $g^x \rightarrow x$ is hard to compute.
    - **Security assumption.**
- Example such groups:
  - Integers modulo a prime. (>1024 bits) (Multiplicative notation! g^x)
  - Points on Elliptic curves. (>160 bits) (Additive notation! xg)

The Benaloh Crypto-system

- First introduced in the context of e-voting, to count votes.
- The Scheme:
  - Public: g, h (and group parameters)
  - Key generation:
    generate a random “x” ($0 < x <$ order of the group);
    Private key is “x”, public key is $pk = g^x$.
  - Encryption of m with pk:
    random k;
    $E(m; k) = (g^k, g^{xk}h^m)$
  - Decryption of (a,b) with x: $m = \log_h(b\,(a^x)^{-1})$ ($= \log_h g^{xk}h^m / g^{xk}$)
- But is $\log_h$ not hard to compute?
  - Make a table for all small (16-32 bit) values.

The additive homomorphism

- Reminder:
  - Encryption: $E(m; k) = (g^k, g^{xk}h^m)$
- Homomorphism
  - Addition of $E(m_0; k_0) = (a_0, b_0)$ and $E(m_1; k_1) = (a_1, b_1)$

    $\mathbf{E(m_0+m_1; k_0+k_1) = (a_0a_1, b_0b_1)}$

    $= (g^{k0}g^{k1} , g^{xk0}h^{m0}g^{xk1}h^{m1}) = (g^{k0+k1}, g^{x(k0+k1)}h^{(m0+m1)}$

  - Multiplication of $E(m_0; k_0) = (a_0, b_0)$ with a constant c:

    $\mathbf{E(cm_0; ck_0) = ((a_0)^c, (b_0)^c)}$
- Not sufficient for all operations. (No multiplication of secrets)

Application 1: Simple Statistics

- Problem: A poll asks a number of participants whether they prefer "red" or "blue". How many said "red" and how many "blue"?

- Solution: Each participant submits a Benaloh ciphertext for both "red" and "blue" to an authority.
  The authority can homomorphically add them.

- Lab 03 will be all about this!

Illustrated

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Compute ...

Alice	Bob	...	Zoe	Total
E(0)	E(1)	...	E(1)	E(10)
E(1)	E(0)	...	E(0)	E(5)

Authority

Discussion

- Domain of plaintext is small (up to number of participants), so decryption by enumeration is cheap.

- The Key questions:
  - Who's public key?
  - Who has the decryption key?
- The Decryption question: Who decrypts?
  - If single entity → TTP!
  - If no-one: scheme is useless! (Outsourced computation?)

Threshold Decryption

- Answer: it is better if no one has the secret key.
  - No TTP!
- Threshold decryption:
  - The secret key is distributed across many different people.
  - Each have to contribute to the decryption.
  - Even if one is missing, remaining cannot decrypt.
- How?
  - Private keys: $x_1, \ldots, x_n$
  - Public key: $g^{x1+\ldots+xn}$
  - Decryption of (a,b): $m = b / a^{x1} / a^{x2} / \ldots / a^{xn}$

Beyond the Benaloh limitations

- Raw RSA:
  - Multiplicative homomorphism
  - No addition :-(

- Paillier Encryption:
  - Additive homomorphism only
  - Based on RSA: large ciphertexts, slow

- Schemes based on Pairings on Elliptic curves:
  - Addition and 1 multiplication!

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- Breakthrough: Gentry (2009) A fully homomorphic scheme
  - Extremely inefficient! But cool.

- Somewhat Homomorphic Schemes:
  - Vinod Vaikuntanathan et al.
  - Larger ciphertexts (30Kb), but fast operations (Add 1ms, Mult 50ms)
  - Variable but limited circuit depth.

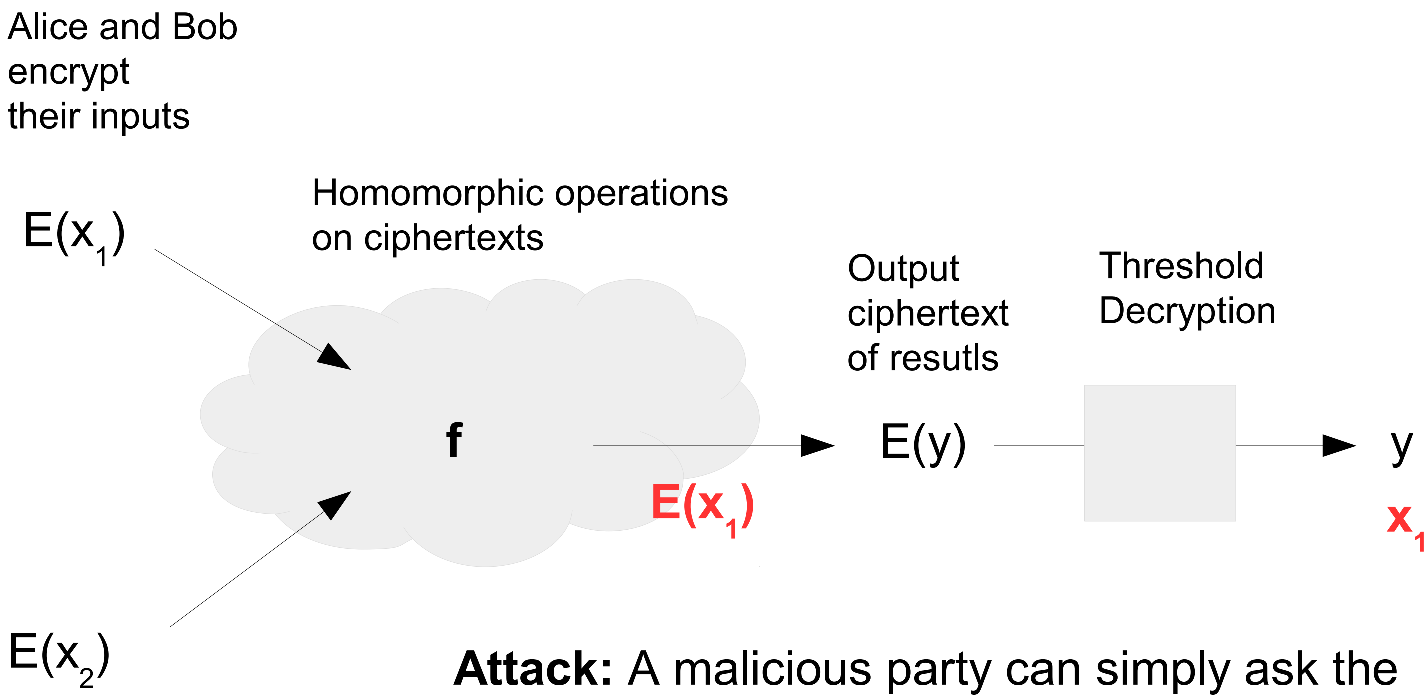
What is cool about homomorphic schemes?

- Simple architecture:
  - Everyone just provides encrypted inputs. One party (any) computes the function.
- Secret functions:
  - Parts of the function itself may remain secret. The service can perform whatever operations without telling any party.
- Powerful and efficient:
  - Any function of shallow depth.
  - Linear operations are very fast. (Order one field multiplications)
  - Multiplications can be fast-ish (for SHE)

The downsides of homomorphisms

- Expressiveness:
  - Expressing computations as boolean circuits makes them much more expensive (example: no binary search!)
- Efficiency:
  - Every bit → 160bit, 1024bits, …, 30Kbs.
- The problem of decryption (Part 2): Integrity

Attack: What is the party doing the computation is actively malicious?



Attack: A malicious party can simply ask the threshold decryption parties to decrypt a secret, not the output of the computation!
(Trade name: a decryption oracle attack)

Lesson: No confidentiality without integrity!

No confidentiality without integrity!

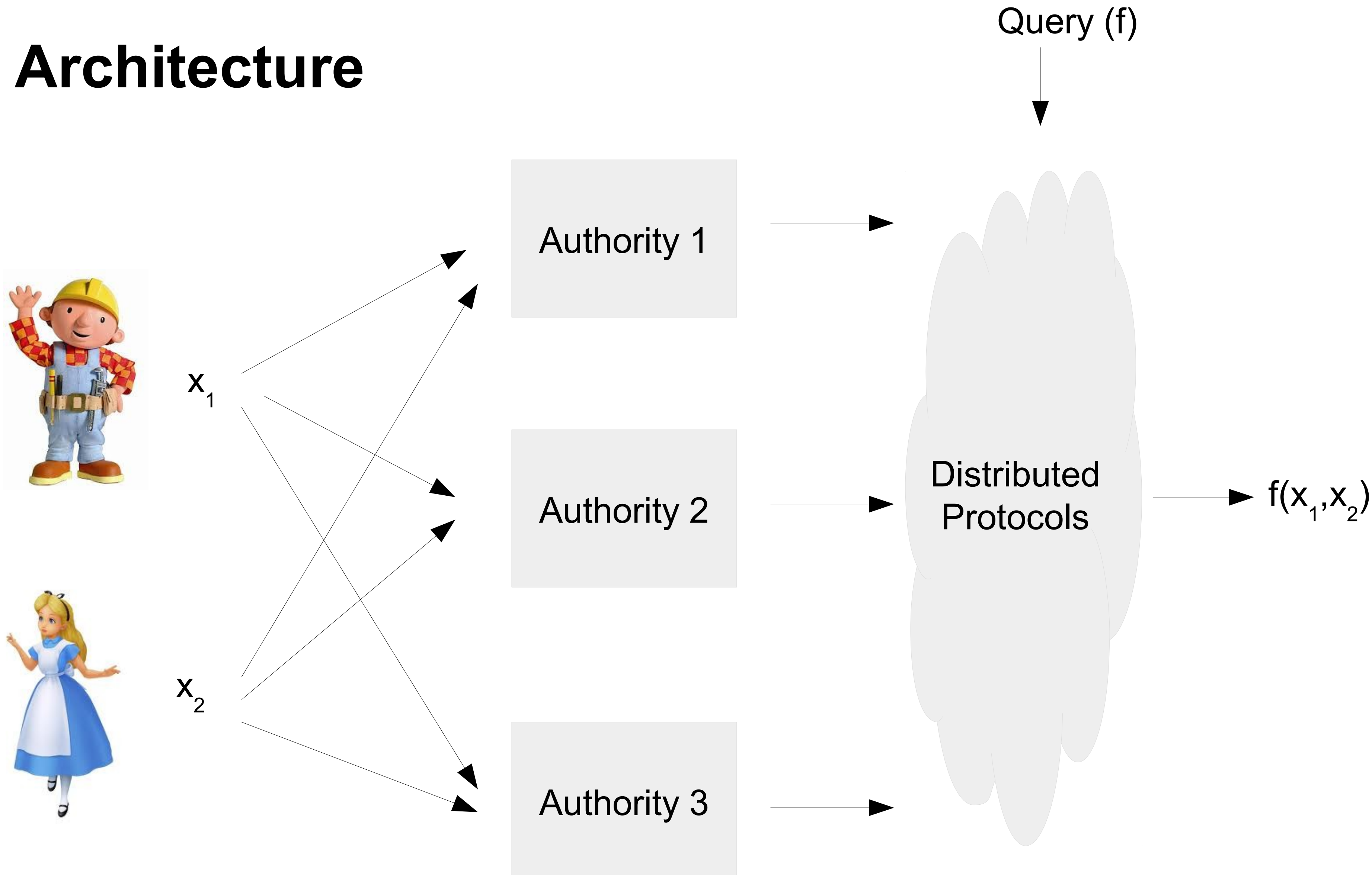
- What to do?
  - The central party needs to prove that the output of the computation was indeed correct.
  - Easy case: computation is public, anyone can verify it
    - Ouch. Expensive.
    - Techniques to verify correctness of outsourced computations.
  - Hard case: computation is private.
    - No one has really dealt with this case.
    - Maybe: if private information can be turned into data? …

Secret sharing

Secret Sharing based private computations

- The core idea:
  - Each secret is “shared” across many authorities.
  - Those authorities use protocols to transform shares of secrets into shares of function of secrets.
  - Key: addition & multiplication
- SPDZ variant:
  - Pre-computations to speed up multiplication (using SHE)
  - Integrity protection, nearly for free!

Architecture



The basic scheme

- We work in the field of integers modulo a prime p
  - Clock arithmetic with “p hour” clock.
- A share of secret “x” is denoted “<x>”
  - If we add all shares “<x>” (mod p) we get “x”
- Toy example:
  - Prime p = 2, x = 1
  - Shares <x> are {1, 1, 0, 1, 0}
  - Check: 1 + 1 + 0 + 1 + 0 mod 2 = 1

Addition of secrets is simple!

- Sharing is based on addition:
  - Natural additive homomorphism.
- Add <a> and <b>:
  - Each authority can simply add the shares
  - <c> = <a+b> = <a> + <b> mod p
  - No distributed protocol is necessary.

Public constant addition and multiplication

- Add <a> to a constant k:
  - Split k into <k> as {0,0,...,0,k}
  - Do addition between <k> and <a>

- Multiply <a> by a public constant k:
  - Each authority privately computes (no interaction)
  - <c> = <ka> = k<a>

Multiplication of secrets

- More complex:
  - Need some pre-computed values.
  - Interactive protocol between authorities.
- Pre-computed values:
  - Independent from the function “f”.
  - Can be batch produced beforehand.
  - How? Using TTP, Secure Hardware, SHE (SPDZ).

Multiplication

- Precomputed triples: <a>, <b>, <c>
  - Such that <c> = <ab>

- Protocol to multiply <x> and <y>:
  - Get fresh pre-computed triplet <a>,<b>,<c>
  - Compute

    <e> = <x> + <a>

    <d> = <y> + <b>
  - Publish <e> and <d> to get e and d.
  - Compute:

    <z> = <xy> = <c> - e<b> - d<a> + ed

Note: a, b are randomly distributed so they totally hide x and y

Linear!

Logic gates

- Share secret input bits <0> or <1>
- Define function f as a circuit
- Boolean gates:
  - NOT(a) = 1 – a
  - AND(a, b) = ab
    - NAND(a, b) = 1 – ab
  - NOR(a, b) = (1 – a) (1 – b)
  - XOR(a, b) = $(a-b)^2$

The problem with circuits

- Doing an addition of a 32 bit number:
  - Multiplicative depth of about 14.
  - Requires many rounds of interaction.
- It is much faster to do linear operations on shares of the actual secrets rather than bits.
- Solution:
  - Protocol to convert shares of bits to full representations.
    eg. <1>, <1> to <3>
  - Protocol to convert a secret share to its bit representation
    eg. <3> to <1>, <1>

Secret Sharing: pros and cons

- Pros:
  - Well understood complete protocols.
  - Actual operations are very cheap.
  - Integrity can be very cheap.
- Cons:
  - Network interactions.
  - Vast number of triplets (one per gate).
  - Complications about generating them.
  - Circuits express inefficiently.
  - Computations cannot be secret!

Overall conclusions

- Private computations:
  - **You can do any computation privately.**
  - **It will cost you.**
    - Compute:homomorphic encryption.
    - Network: secret sharing.
  - Linear operations are cheap.
  - Non-linear operations less so.
  - Limited non-linear depth helps a lot with efficiency.

- Integrity:
  - A problem for confidentiality.

- Maturity:
  - Tool chains and compilers: research grade.
  - Too slow to use for bulk computations.
  - Special high-value computations OK – i.e. billing.
  - Use it to implement functions of the TCB securely.